



CMA Quantum Technologies QuantEdu-France Project

Doctoral contracts

Thesis supervision

Thesis title	Advanced Symmetry Reduction in Quantum Information
Research unit	LaBRI
Doctoral School of affiliation	EDMI
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International collaboration <i>(If applicable, please specify)</i>	Co-supervised doctorate (cotutelle) <input type="checkbox"/> Co-supervision <input type="checkbox"/> None <input checked="" type="checkbox"/>

Project summary

For online publishing and dissemination purposes, in a format that can be integrated into the platforms: adum, EURs

The field of quantum information is awash with difficult large-scale optimization problems. However, the sheer size of the matrices involved makes – for all practical purposes – their numerical solution impossible. Recent years have seen symmetry reduction techniques attacking such problems with some success, compressing large matrices to a collection of smaller ones. Yet progress is hampered by a lack of integration of analytical and numerical methods, and by the presence of obstructions to an optimization formulation. Advanced symmetry reduction in quantum information (ASQI) aims to radically rethink current approaches to hard mathematical optimization in quantum information, by putting an equal focus on suitable problem formulations in combination with the integrated use of state-of-the-art reduction techniques. First, we systematically integrate powerful reduction techniques: analytical symmetry reduction through the underlying algebras, numerical dimension reduction with Jordan algebras, sparse optimization that simplifies matrices with many zeros, SDP sketching, and approximation algorithms. The computational boost will allow to attack long-standing open problems numerically, providing efficient entangled approximations to quantum Max Cut and to bound the number of mutually unbiased bases in dimension six. Second, we develop novel formulations of structured problems in terms of mathematical optimization. We focus on two problems: the quantum capacity of quantum graphs and Bell inequality violations in network scenarios. These pose unique obstructions to optimization such as shifted factorizations

and nonlinearity in the state. To overcome these, we take advantage of novel optimization frameworks such as equivariant, trace, and state polynomial optimization. ASQI renders several long-standing open quantum information problems amenable to numerical techniques and provides systematic recipes for formulating problems in the optimization framework and their symmetry reduction, delivering impact across fields.

Detailed project

Scientific description of the project

Quantum computation aims to perform calculations with quantum systems, for example atoms or photons. But understanding what happens inside quantum computers presents huge mathematical challenges. Many of these are in the area of optimization, maximizing or minimizing some quantity of interest. It is only natural that methods from mathematical optimization, such as semidefinite (SDP) and conic programming, able to give certificates of optimality, have now become workhorses for studying quantum behavior. Yet, two key issues are hampering progress:

a) There are problems whose associated Hilbert space dimension is simply too large. An example is the quantum Max Cut problem, where one aims to determine the ground state energy of the Heisenberg Hamiltonian on a graph. Recent work has focused on finding approximations, but entangled approximations can't go beyond a dozen qubits. Yet for understanding frustration in quantum many-body systems, one would like to solve this on graphs consisting of hundreds if not thousand vertices. Another example, important in quantum tomography, communication, and cryptography, is the maximal number of mutually unbiased bases in dimension six. The task is to determine the maximum number of orthonormal bases whose mutual fidelity is $1/6$. It is widely believed (but unproven) that the number of such bases is three. Despite being well-known problems with ready formulations in terms of mathematical optimization, their size keeps them out of reach of current approaches.

b) Some long-standing open problems in quantum information are about maximizing or minimizing a certain quantity. Yet, no closed-form formulation in terms of mathematical optimization is known. Examples are the quantum analog of the famous Shannon capacity of graphs and Bell inequality violations in network scenarios. These problems feature certain obstructions to known methods such as non-commutative optimization: non-linearity in the state and factorization constraints over shifted partitions. However, related problems hint towards mathematical optimization formulations.

The strategic question is: How can we exploit the recent wealth in analytical and numerical methods to make fast progress on key questions in quantum information? Our answer is:

A) to integrate analytical and numerical symmetry reduction methods;

B) to overcome obstructions to mathematical optimization.

Both A) and B) are based on the rationale that recent progress in mathematical optimization makes it imperative to address several hard problems in quantum information now. These developments include the recent equivariant, trace, and state polynomial optimization frameworks; dimension reduction with Jordan algebras; sparsity and cluster methods; diagonally dominant sum-of-squares;

approximation algorithms; and the representation theory of symmetric group wreath products, Brauer, and Terwilliger algebras. These tools combined allow for dealing with unconventional optimization problems while providing a radical reduction in problem size. It is the ambition of ASQI to provide the community with practical recipes for dealing with structured large-scale problems: this includes strategies for identifying invariant formulations, the choice of reduction techniques, and identifying trade-offs incurred from approximations. ASQI also widens the applicability of optimization frameworks by systematically resolving obstructions to optimization: non-linear state expressions, dimension-freeness, and “shifted” tensor factorizations.

$$\begin{pmatrix} M_{11} & & M_{1n} \\ & \ddots & \\ M_{n1} & & M_{nn} \end{pmatrix} \xrightarrow{\text{Symmetry reduction}} \begin{pmatrix} A_{11} & & A_{1j} \\ & \ddots & \\ A_{j1} & & A_{jj} \end{pmatrix} \begin{pmatrix} B_{11} & & B_{1k} \\ & \ddots & \\ B_{k1} & & B_{kk} \end{pmatrix} \begin{pmatrix} C_{11} & & C_{1\ell} \\ & \ddots & \\ C_{\ell 1} & & C_{\ell\ell} \end{pmatrix}$$

FIG. 1. Symmetry reduction reduces the size of a large matrix by representing it through a collection of smaller matrices, without losing information of the problem.

PHD OBJECTIVES

Objectives A.1 and A.2 below integrate reduction methods, while objectives B.1 and B.2 overcome obstructions to optimization.

□ **A.1 Quantum Max Cut.** Provide a second order cone program approximation algorithm for quantum Max Cut and mixed Max-Cut/EPR Hamiltonian instances on 100 – 1000 qubits. The algorithm is invariant with tuneable approximation, providing theoretical guarantees.

□ **A.2 Mutually unbiased bases (MUB).** Formulate an invariant relaxation for the MUB problem and an semi-definite programming (SDP) infeasibility sketch. Implement a numerical pipeline integrating dimension reduction and sparsity techniques. Determine the maximal number of mutually unbiased bases in dimension six.



FIG. 2. Left: The antiferromagnetic Heisenberg Hamiltonian on a graph is also known as quantum Max Cut and shows the typical frustration on graphs that are not bipartite. Right: The MUB problem seeks a collection of orthogonal bases, such that the fidelity between basis vectors of different bases is $\frac{1}{d}$.

□ **B.1 Quantum capacity of quantum graphs.** Formulate complete hierarchies for maximal quantum code size and quantum capacity of quantum channels. Relate to the Lovász number bound and state a numerically useful symmetry-reduction. Resolve the obstruction of nonlinear state expressions.

□ **B.2 Network nonlocality.** Provide a linear-programming relaxation on Bell inequality violations in large-scale network nonlocality scenarios. Provide a complete SDP hierarchy to bound the non-local correlations in triangle and network scenarios. Solve the obstruction of shifted tensor factorizations.

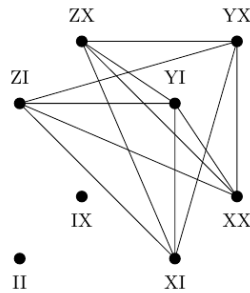


FIG. 3. The confusability graph of a subset of two-qubit errors. Vertices are connected by an edge if the corresponding operators anti-commute. What is the capacity of the corresponding quantum channel and maximal size of a corresponding quantum error correcting code? ASQII provides a closed-form formulation in terms of mathematical optimization.

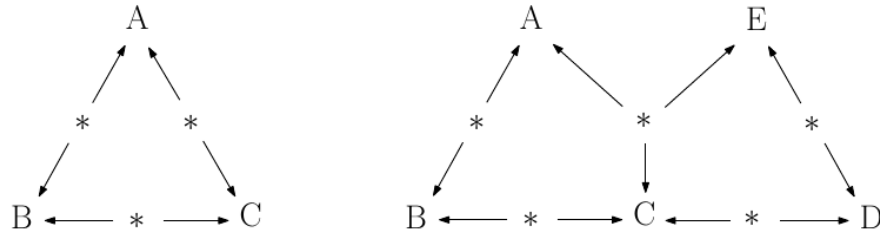


FIG. 4. Left: the triangle network scenario, where three parties share three bipartite sources. The question is, what maximal quantum violation of a given Bell inequality can the parties achieve? Right: a more complicated network scenario. No closed-form formulations in terms of a mathematical optimization problem is known.

SUPPORT FOR THE CANDIDATE

The candidate will join the recently created « Quantum Information and Computation » research group of LaBRI: <https://quantique.labri.fr/>

She/he will thus fully benefit from the LaBRI scientific environment. Moreover, the candidate will benefit from the ANR funding associated to Felix Huber’s CPJ « Algorithmique quantique ». Finally, the candidate will benefit for additional funding (travelling, conference attendance *etc.*) via the PEPR EPIQ - « Quantum Software » that LaBRI is part of.

Project justification

Explain (in a few lines) how the thesis topic fits within the scientific scope of QuantEDu-France.

The proposed PhD thesis topic lies within the framework of quantum information. Quantum information theory encompasses concepts from computers science, but also from physics, mathematics, and engineering, thus offering opportunities for groundbreaking research. Moreover, quantum information is one of the pillars, with quantum technologies, quantum algorithmics and so, of the QuantEdu-France project.

Collaborations

Describe (in a few lines) any academic, industrial and/or international collaborations in which the thesis subject is involved.

Academic collaborations:

The thesis subject can naturally led to an academic collaboration with Ion Nechita (CNRS Toulouse), who is an expert on quantum information and which has recently starting working on quantum graphs, one of the topics of our project (see above). Let us mention here that Ion Nechita has been a visitor of LaBRI on various occasions in the past years.

Industrial collaborations:

The « Quantum Information and Computation » LaBRI research group is currently collaborating with the newly created IBM Quantum France research group, via various collaborations with Simon Martiel, the IBM Quantum France group leader. This PhD thesis could thus lead to new collaborations with this recently created group.