

A Quantum Programming Language for Coherent Control

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Elements of the control structure:

- **if** statements
- **while** statements
- Iteration

An example of quantum control: `qcase`

The operation `qcase` acts on a pair of qubits (“control” and “target”).
For U and V unitary:

- If the control qubit is in state $|0\rangle$, apply U to the target qubit.
- If the control qubit is in state $|1\rangle$, apply V to the target qubit.
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The goal is to develop a programming language that supports coherent control.

Main difficulties

Combining qcase with...

- ① **Measurement**
- ② **Recursion**

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② Recursion

- Incompatibility of qcase with the Löwner ordering.

Costin Bădescu and Prakash Panangaden. “Quantum alternation: Prospects and problems”. In: *12th International Workshop on Quantum Physics and Logic* (2015)

A programming language for quantum control

Syntax

The syntax is inspired by the programming language QPL.

Statements

$$\begin{aligned}
 S, S_0, S_1 ::= & \text{skip} \mid \text{new qbit } q \mid \text{discard } q \mid q * = U \mid S_0; S_1 \\
 & \mid \text{meas } q \ (0 \rightarrow S_0, 1 \rightarrow S_1) \mid \text{while } q \text{ do } S \\
 & \mid \text{qcase } q \ (0 \rightarrow S_0, 1 \rightarrow S_1)
 \end{aligned}$$

where U is a single-qubit unitary map.

Peter Selinger. "Towards a quantum programming language". In: *Mathematical Structures in Computer Science* 14.4 (2004), pp. 527–586

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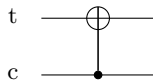
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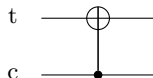
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- **meas** $q (0 \rightarrow S_0, 1 \rightarrow S_1)$ the qubit q , and executes S_0 or S_1 depending on the outcome.
- **while** q **do** S is a loop where q is measured at every iteration.
- **qcase** $q (0 \rightarrow S_0, 1 \rightarrow S_1)$ implements quantum control: S_0 and S_1 are executed in superposition depending on the state of q .

Example: CNOT



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$S_{\text{CNOT}} := \mathbf{qcase} \ c \ (0 \rightarrow \mathbf{skip}, 1 \rightarrow t * = X)$

where $X := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Operational semantics

Defining an operational semantics

The goal is to define the evolution of $|\psi\rangle$ induced by a program statement S :

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A transition occurs with probability $\frac{\| |\psi'\rangle \|^2}{\| |\psi\rangle \|^2}$.

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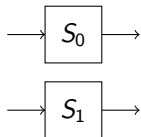
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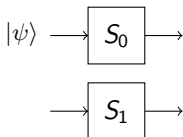
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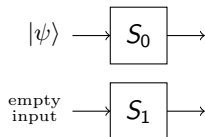
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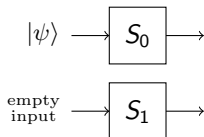
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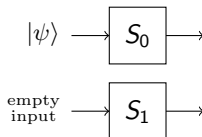


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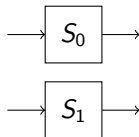
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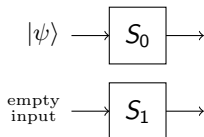
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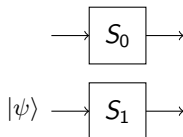
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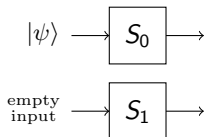
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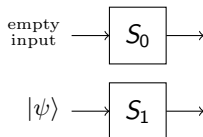
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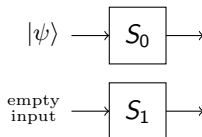
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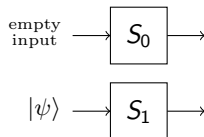
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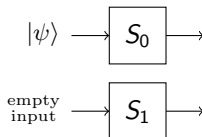


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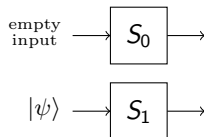
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Therefore, we define default transitions.

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Transition system

Transitions have the form

$$[S, |\psi\rangle] \xrightarrow{\nu} |\psi'\rangle$$

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Denotational semantics

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The goal is to assign to each program a mathematical operation that describes its behavior.

- First idea: Interpret programs as completely positive trace non-increasing maps:

$$\llbracket P \rrbracket = \mathcal{C}$$

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This is not possible: $\text{QCASE}(\mathcal{C}, \mathcal{D})$ is ill-defined.

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Vacuum-extended operations

A vacuum extension of a quantum operation \mathcal{C} is a quantum operation $\tilde{\mathcal{C}}$ such that:

- On regular states: $\tilde{\mathcal{C}}(\rho) = \mathcal{C}(\rho)$.
- On the vacuum state: $\tilde{\mathcal{C}}(|\text{vac}\rangle\langle\text{vac}|) = |\text{vac}\rangle\langle\text{vac}|$.

Hlér Kristjánsson et al. “Resource theories of communication”. In: *New Journal of Physics* 22.7 (2020), p. 073014

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$\text{QCASE}((\mathcal{C}, F), (\mathcal{D}, G))$ is well defined.

$$\text{QCASE}((\mathcal{C}, F), (\mathcal{D}, G)) = \left(\begin{bmatrix} \rho_1 & \rho_2 \\ \rho_3 & \rho_4 \end{bmatrix} \mapsto \begin{bmatrix} \mathcal{C}(\rho_1) & F\rho_2G^\dagger \\ G\rho_3F^\dagger & \mathcal{D}(\rho_4) \end{bmatrix}, \begin{bmatrix} F & 0 \\ 0 & G \end{bmatrix} \right)$$

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Some statement interpretations:

$$\llbracket \text{new qbit } q \rrbracket := (|0\rangle\langle 0|_q \otimes -, |0\rangle_q \otimes I_\Gamma)$$

$$\llbracket q * = U \rrbracket := (U_q(\cdot)U_q^\dagger, U_q)$$

$$\llbracket \text{qcase } q (0 \rightarrow P, 1 \rightarrow Q) \rrbracket := \text{QCASE}_q[\llbracket P \rrbracket, \llbracket Q \rrbracket]$$

$$\llbracket \text{while } q \text{ do } S \rrbracket := \text{lfp}(\mathcal{F}_q^S)$$

where $\mathcal{F}_q^S(\mathcal{C}, F) := \overline{\text{meas}}_q[(\mathcal{I}, I), (\mathcal{C}, F) \circ \llbracket S \rrbracket]$.

Results

Universality and adequacy

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Adequacy

The operational and denotational semantics describe the same behavior of programs.

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S and S' are *observationally equivalent* ($S \approx S'$) if their behavior is indistinguishable in any context.

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$$\begin{aligned} \mathbf{qcase} \ q \ (0 \rightarrow S_0, 1 \rightarrow S_1); \mathbf{qcase} \ q \ (0 \rightarrow S'_0, 1 \rightarrow S'_1) \\ \approx \\ \mathbf{qcase} \ q \ (0 \rightarrow S_0; S'_0, 1 \rightarrow S_1; S'_1) \end{aligned}$$

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S and S' are *observationally equivalent* ($S \approx S'$) if their behavior is indistinguishable in any context.

Full abstraction

$$S \approx S' \quad \text{iff} \quad \llbracket P \rrbracket = \llbracket Q \rrbracket$$

Exemples :

new qbit q ; **discard** $q \approx \text{skip}$

$$\begin{aligned} \text{qcase } q (0 \rightarrow S_0, 1 \rightarrow S_1); \text{qcase } q (0 \rightarrow S'_0, 1 \rightarrow S'_1) \\ \approx \\ \text{qcase } q (0 \rightarrow S_0; S'_0, 1 \rightarrow S_1; S'_1) \end{aligned}$$

qcase $q (0 \rightarrow S, 1 \rightarrow S) \not\approx S$ in general

Conlusion

Conclusion

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Questions?