Alternative adiabatic quantum dynamics
with algorithmic applications

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$$H(0)$$

Adiabatic quantum computing

$$H(0) \quad \longrightarrow \quad H(1)$$

Adiabatic quantum computing

- Physically motivated

Adiabatic quantum computing

- Physically motivated
- Natively supported

Adiabatic quantum computing

- Physically motivated
- Natively supported
- Serves as inspiration for circuit algorithms

Adiabatic quantum computing

- Can this setup be ported to a circuit based quantum computer?

Adiabatic quantum computing

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- More flexibility!

Rescaling time in the Schrödinger equation

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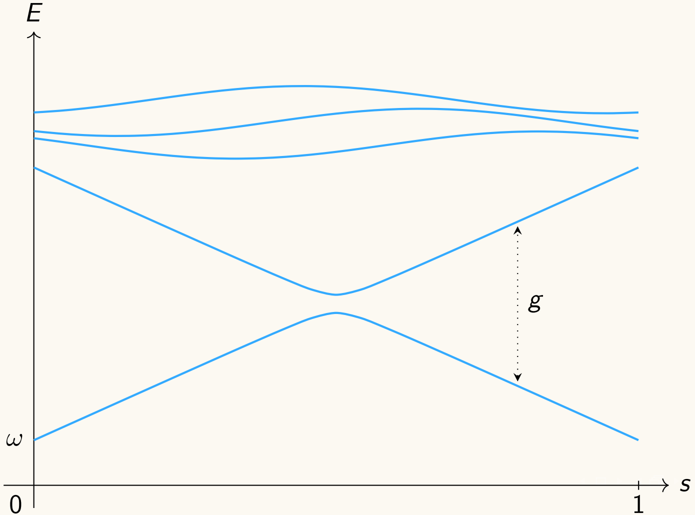
$$\frac{\mathrm{d}|\psi\rangle}{\mathrm{d}s} = -\frac{i}{\hbar} TH|\psi\rangle.$$

The gap of a path of Hamiltonians

Let $H(s)$ be a Hamiltonian for all $s \in [0, 1]$.



A baseline result

Theorem

Let $H(s)$ be a twice continuously differentiable time-dependent Hamiltonian. Let $U(s)$ be generated by $-iTH(s)$. Then

$$\sqrt{1-F} \leq \frac{1}{T}\frac{\|H'\|}{g^2}\Big|_{s=0} + \frac{1}{T}\frac{\|H'\|}{g^2}\Big|_{s=1} + \frac{1}{T}\int_0^1 \left(\frac{\|H''\|}{g^2} + (2\sqrt{2}+1)\frac{\|H'\|^2}{g^3}\right) \mathrm{d}s.$$

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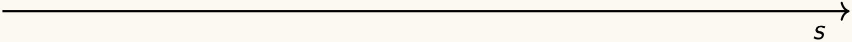
$$T = O\left(\frac{\|H'\|^2}{g_m^3} + \frac{\|H''\|}{g_m^2}\right)$$

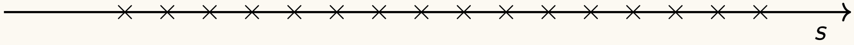
$$\mathcal{T} e^{-i\int_0^1 H_s \,\mathrm{d}s} = \prod_{k=0}^{n} e^{\frac{-i}{n} H(\frac{k}{n})} + O\left(\frac{T^2}{n}\right).$$

Poisson-distributed dephasing

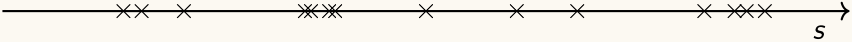
Poisson processes



Poisson processes



Poisson processes



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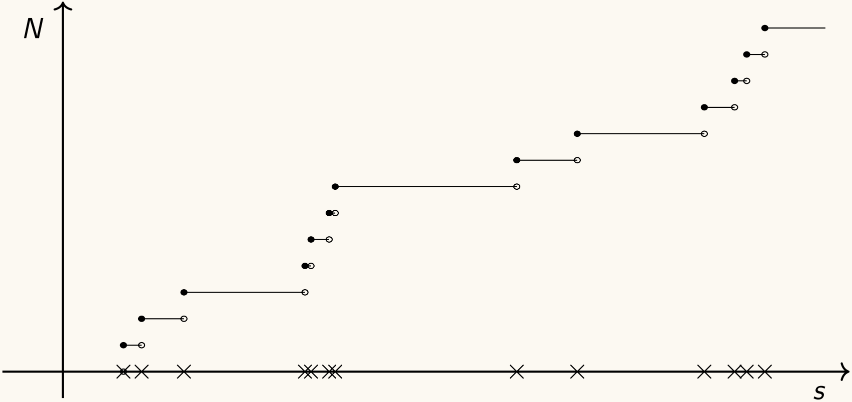

Deriving dynamics: unitaries

We need to consider classical randomness: $|\psi\rangle \to \rho$

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$$\frac{\mathrm{d}\rho}{\mathrm{d}s} = \lambda\Big(U(s)\rho U(s)^* - \rho\Big).$$

An adiabatic theorem

Theorem

Let $\rho(s)$ *be the solution of the differential equation*

$$\frac{\mathrm{d}\rho}{\mathrm{d}s} = \lambda \mathcal{L}_s(\rho).$$

Suppose $P(s)$ *is the ground state projector and*

- $\mathrm{Tr}\left(P(0)\rho(0)\right) = 1$;
- $\mathrm{Tr}(P(s)Y) = 0$ *for all* Y *in the image of* $\mathcal{L}_s$;
- *there exists* $X(s)$ *such that* $P'(s) = \mathcal{L}_s^*\big(X(s)\big)$ *for all* $s \in [0,1]$.

Then

$$1 - F \le \left.\frac{\|X\|}{\lambda}\right|_{s=0} + \left.\frac{\|X\|}{\lambda}\right|_{s=1} + \int_0^1 \left(\frac{\|X'\|}{\lambda} + \left|\left(\frac{1}{\lambda}\right)'\right| \|X\|\right) \mathrm{d}s.$$

Discrete adiabatic theorem

For $\mathcal{L}_s(\rho) = U(s)\rho U(s)^* - \rho$, we can take

$$X = P'(\mathbb{1} - \omega U^*)^+ + (\mathbb{1} - \overline{\omega} U)^+ P'$$

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Theorem

Let $U(s)$ be a norm-continuous bounded time-dependent unitary operator that is twice continuously differentiable. Then

$$1 - F \leq \frac{\|U'\|}{\lambda g^2}\Big|_{s=0} + \frac{\|U'\|}{\lambda g^2}\Big|_{s=1} + 2\int_0^1 \left(\frac{\|U''\|}{\lambda g^2} + 8\frac{\|U'\|^2}{\lambda g^3} + \frac{\|U'\|}{g^2}\right) \mathrm{d}s.$$

Choosing the unitary U

Qubitisation

$$U(s) = \begin{pmatrix} H(s) & -\sqrt{\mathbb{1} - H(s)^2} \\ \sqrt{\mathbb{1} - H(s)^2} & H(s) \end{pmatrix}$$

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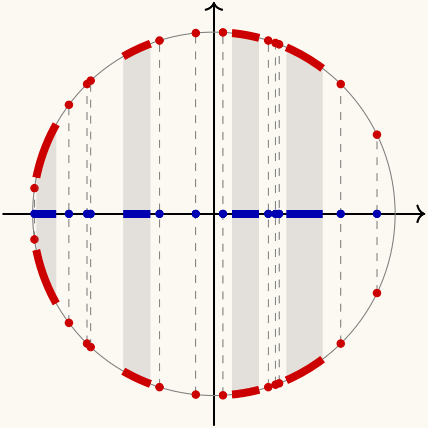

$\sigma(H)$

$\sigma\begin{pmatrix} H & -\sqrt{\mathbb{1} - H^2} \\ \sqrt{\mathbb{1} - H^2} & H \end{pmatrix}$

Qubitisation

Theorem

The infidelity under Poissonnised qubitisation is bounded by

$$1 - F \leq \left.\frac{2\|H'\|}{\sqrt{1-\omega^2}\lambda g^2}\right|_{s=0} + \left.\frac{2\|H'\|}{\sqrt{1-\omega^2}\lambda g^2}\right|_{s=1} + 2\int_0^1 \left(\frac{2\|H'\|}{\sqrt{1-\omega}^{3/2}\lambda g^2} + \frac{\|H'\|^2 + \|H''\|}{\sqrt{1-\omega^2}\lambda g^2} + \frac{32}{1-\omega^2}\frac{\|H'\|^2}{\lambda g^3}\right) \mathrm{d}s.$$

Time-independent Hamiltonian evolution

Theorem

The infidelity under Poissonnised unitary evolution with $U(s) = e^{-ihH(s)}$ is bounded by

$$1 - F \leq 2\frac{\|H'\|}{\lambda h g^2}\bigg|_{s=0} + 2\frac{\|H'\|}{\lambda h g^2}\bigg|_{s=1} + 4\int_0^1 \left(\frac{\|H''\|}{\lambda h g^2} + 8\frac{\|H'\|}{\lambda h g^3}\right) \mathrm{d}s$$

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$\mathcal{T} e^{-i\int_0^1 H(s)\,\mathrm{d}s} \approx \prod_{k=0}^{n} e^{-i\frac{1}{n}H(\frac{k}{n})}$

Trotter step

Now assume $H_s = (1-s)H_0 + sH_1$.

Theorem

The infidelity under Poissonnised unitary evolution with $U(s) = e^{-ih(1-s)H_0} e^{-ihsH_1}$ *is bounded by*

$$1 - F \leq 4 \frac{\|H_0 - H_1\|}{\lambda h (g - h/2)^2}\bigg|_{s=0} + 4 \frac{\|H_0 - H_1\|}{\lambda h (g - h/2)^2}\bigg|_{s=1} + 8 \int_0^1 \left(\frac{\|H_0^2 - H_1^2\|}{\lambda (g - h/2)^2} + 32 \frac{\|H_0 - H_1\|^2}{\lambda h (g - h/2)^3} \right) \mathrm{d}s$$

Phase randomisation

$$\frac{\mathrm{d}\rho}{\mathrm{d}s} = \lambda\Big(\int_{-\infty}^{\infty} e^{-i\tau H_s}\rho e^{i\tau H_s}\,\mathrm{d}\mu(\tau) - \rho\Big)$$

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Theorem

The infidelity is bounded by

$$1 - F \leq \frac{\|H'\|}{\lambda g}\Big|_{s=0} + \frac{\|H'\|}{\lambda g}\Big|_{s=1} + \int_0^1 \Big(\frac{\|H''\|}{\lambda g} + 4\frac{\|H'\|^2}{\lambda g^2}\Big)\,\mathrm{d}s$$

Procedure	Adiabatic theorem
Phase randomisation	$1 - F \leq \frac{\|H'\|}{\lambda g}\Big\|_{b.c.} + \int_0^1 \left(\frac{\|H''\|}{\lambda g} + 4\frac{\|H'\|^2}{\lambda g^2}\right) \mathrm{d}s$
Discrete adiabatic (general U)	$1 - F \leq \frac{\|U'\|}{\lambda g^2}\Big\|_{b.c.} + 2\int_0^1 \left(\frac{\|U''\|}{\lambda g^2} + 8\frac{\|U'\|^2}{\lambda g^3}\right) \mathrm{d}s$
Qubitised U	$1 - F \leq \frac{2\|H'\|}{\sqrt{1-\omega^2}\lambda g^2}\Big\|_{b.c.} + 2\int_0^1 \left(\frac{2\|H'\|}{\sqrt{1-\omega}^{3/2}\lambda g^2} + \frac{\|H'\|^2 + \|H''\|}{\sqrt{1-\omega^2}\lambda g^2} + \frac{32}{1-\omega^2}\frac{\|H'\|^2}{\lambda g^3}\right) \mathrm{d}s$
Time-independent evolution	$1 - F \leq 2\frac{\|H'\|}{\lambda h g^2}\Big\|_{b.c.} + 4\int_0^1 \left(\frac{\|H''\|}{\lambda h g^2} + 8\frac{\|H'\|}{\lambda h g^3}\right) \mathrm{d}s$
Trotter step	$1 - F \leq 4\frac{\|H_0 - H_1\|}{\lambda h (g - h/2)^2}\Big\|_{b.c.} + 8\int_0^1 \left(\frac{\|H_0^2 - H_1^2\|}{\lambda (g - h/2)^2} + 32\frac{\|H_0 - H_1\|^2}{\lambda h (g - h/2)^3}\right) \mathrm{d}s$
Adiabatic time-evolution	$1 - F \leq \frac{\|H'\|}{\lambda g^2}\Big\|_{b.c.} + \int_0^1 \left(\frac{\|H''\|}{\lambda g^2} + 5\frac{\|H'\|^2}{\lambda g^3}\right) \mathrm{d}s$

Conclusion

- Four discrete procedures for implementing adiabatic dynamics

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- If we just want to track the eigenstate, the discretisation cost scales linearly in time (cfr. [Yi21], [AnCostaBerry25])

Open questions

- Other operations / unitaries?

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- Different paths of Hamiltonians?

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- Different paths of Hamiltonians?
- Integrators other than Poisson processes (e.g. noise)

Bibliography

[Yi21] Changhao Yi. 'Success of Digital Adiabatic Simulation with Large Trotter Step'. *Physical Review A*, vol. 104, no. 5, Nov. 2021, p. 052603.

[...aBerry25] Dong An, Pedro C. S. Costa and Dominic W. Berry. 'Large time-step discretisation of adiabatic quantum dynamics'. `https://arxiv.org/abs/2509.00171v1`.