Optimal Untelegraphable Encryption

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JIQ 26 – Bordeaux

January 16, 2026

No-cloning & no-telegraphing theorems

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No-telegraphing theorem (Werner, 1998)

Arbitrary quantum state cannot be transmitted through classical channels without pre-shared entanglement.

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However, they are not computationally equivalent (Nehoran and Zhandry, 2024).

Uncloneable encryption

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Encryption Scheme

Generation: $\mathsf{Gen}(\mathsf{security}) = \mathsf{key}$

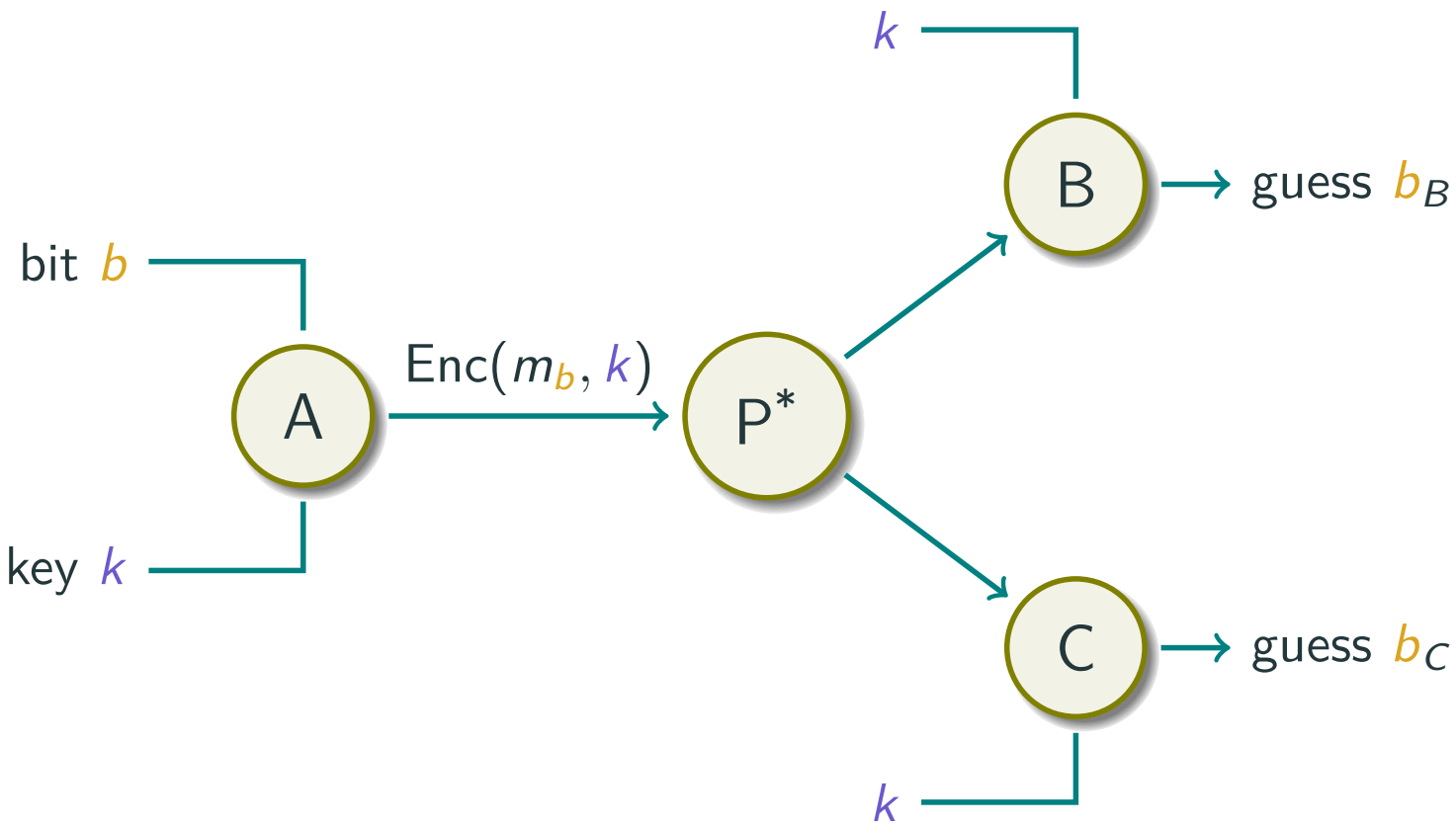
Encryption: $\mathsf{Enc}(\mathsf{message}, \mathsf{key}) = \mathsf{ciphertext}$

Decryption: $\mathsf{Dec}(\mathsf{ciphertext}, \mathsf{key}) = \mathsf{message}$

Security is defined against a cloning attack: a single quantum ciphertext is processed once and then used to enable two separated parties, both holding the key, to recover information about the message.

Untelegraphable encryption

As uncloneable encryption, but restricted to telegraphing attacks:



Goal: Prevent parties B and C from simultaneously guessing bit b with high probability.

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*P is a quantum-to-classical CPTP map.

Haar-measure encryption

For a $\log(n)$-bit message $m \in [n]$ and a Haar-random unitary $U \in \mathrm{U}(d)$ as the key:

$$\mathrm{Enc}(m, U) = U\Big(\underbrace{|m\rangle\langle m|}_{n\times n \text{ matrix}} \otimes \underbrace{I_{d/n}}_{\text{identity}} \Big) U^*$$

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Efficiency (plain model vs. computational model)

Sampling a Haar-random U is not efficient.

Plain model $\Rightarrow$ unitary t-design (bounded moments security analysis).

Computational model $\Rightarrow$ pseudorandom unitary.

Result 1

Untelegraphable-indistinguishability

The Haar-measure encryption scheme for classical bits (2 messages) achieves untelegraphable-indistinguishable security, with telegraphing attack success probability upper bounded by

$$\frac{1}{2} + \underbrace{\frac{1}{2\sqrt{d+1}}}_{\text{negligible}}.$$

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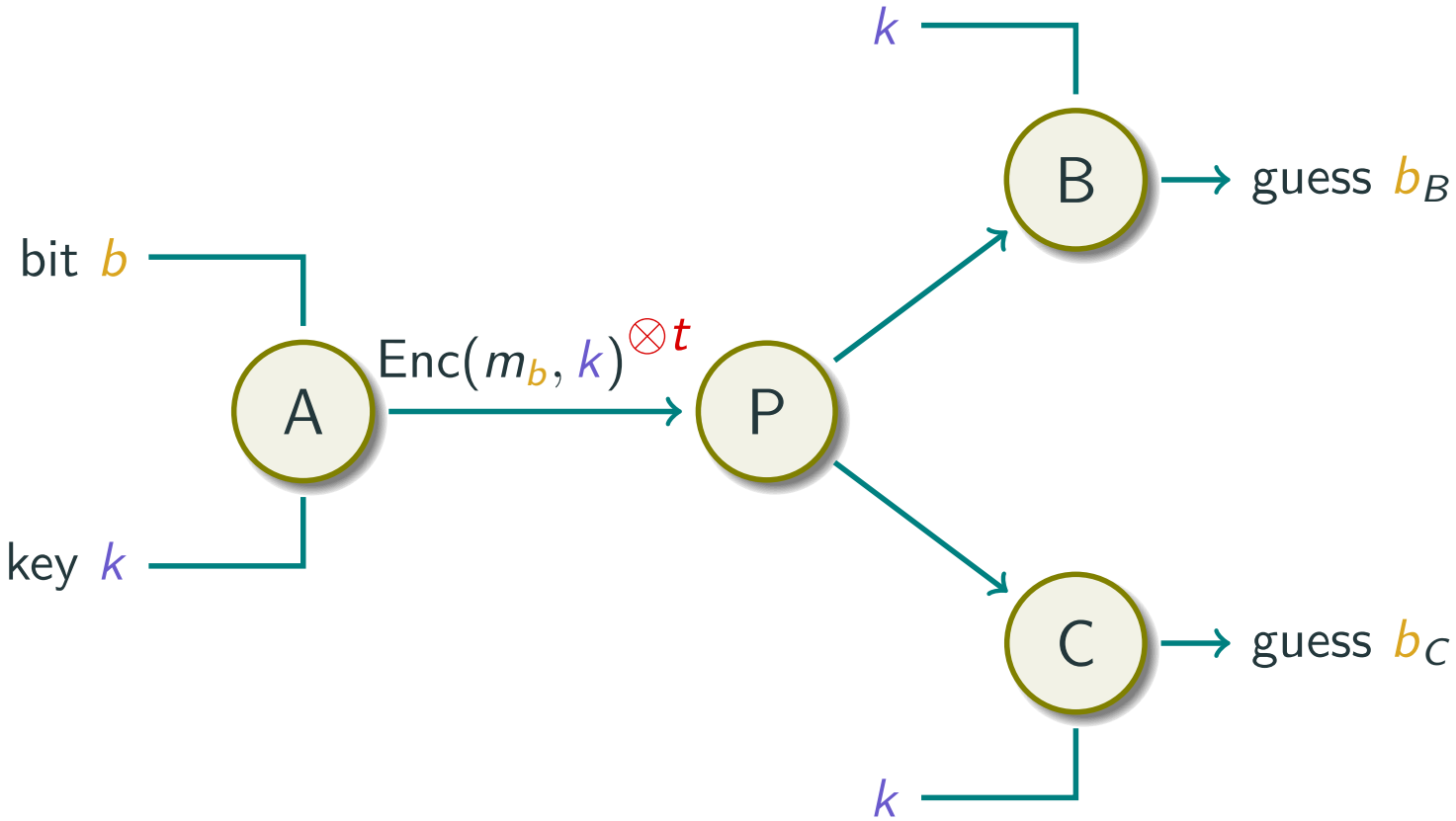
In contrast, the best known upper bound for cloning attack is (Bhattacharyya and Culf, 2025):

$$\frac{1}{2} + \underbrace{\frac{3\log\log d}{2\log d}}_{\text{not negligible}}.$$

t-copy untelegraphable encryption

Unlike uncloneable encryption, untelegraphable encryption admits a stronger adversarial model where P receives t copies of the ciphertext.



Result 2

t-copy untelegraphable-indistinguishability

The Haar-measure encryption scheme for n classical messages achieves t-copy untelegraphable-indistinguishable security, with telegraphing attack success probability upper bounded by

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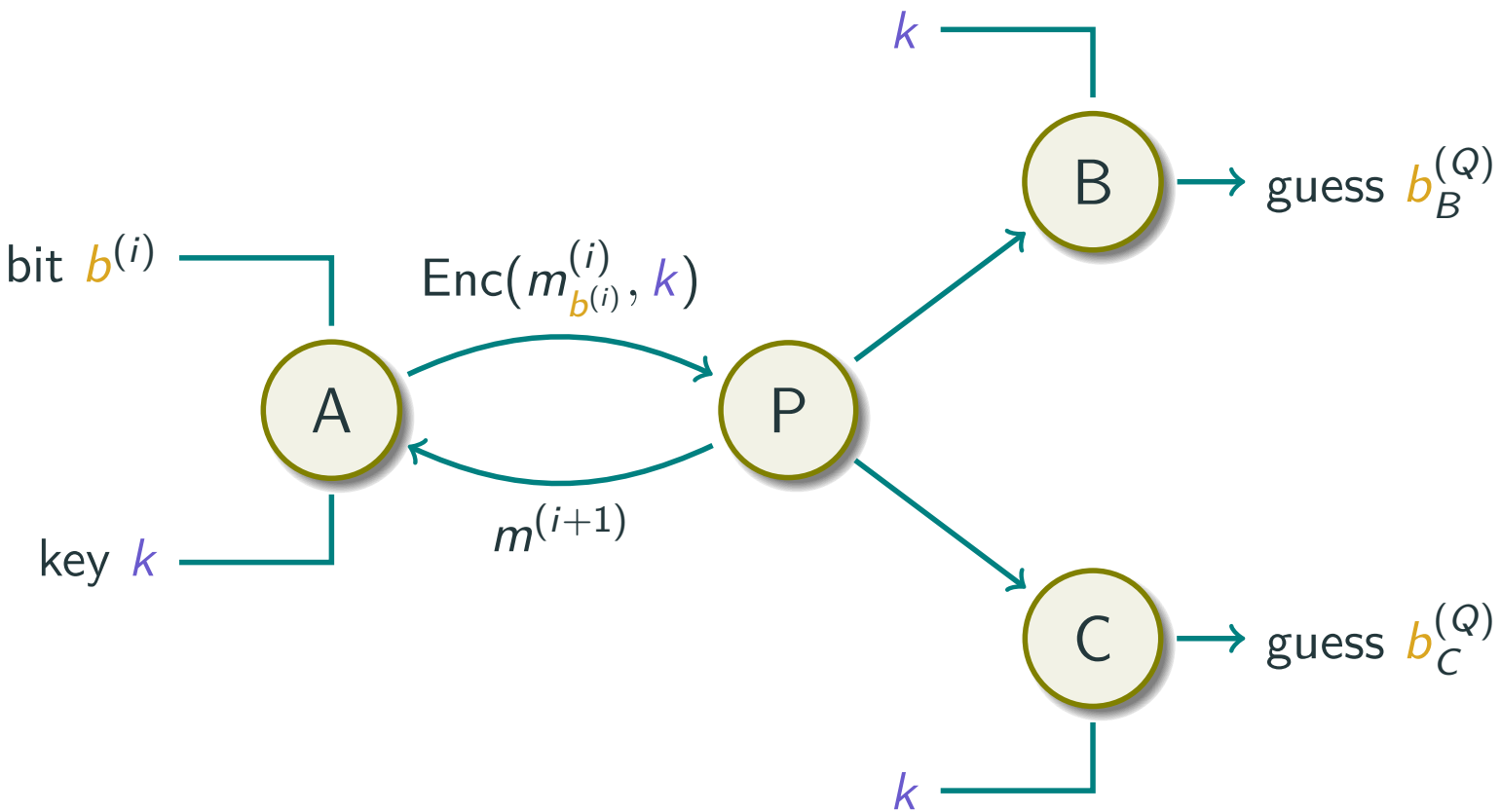
The proof relies on Haar moments up to order $2t$, and therefore requires a unitary $2t$-design.

Collusion-resistant untelegraphable encryption

In a collusion attack, the adversary P adaptively interacts with the sender A across successive Q rounds.



Result 3 & 4

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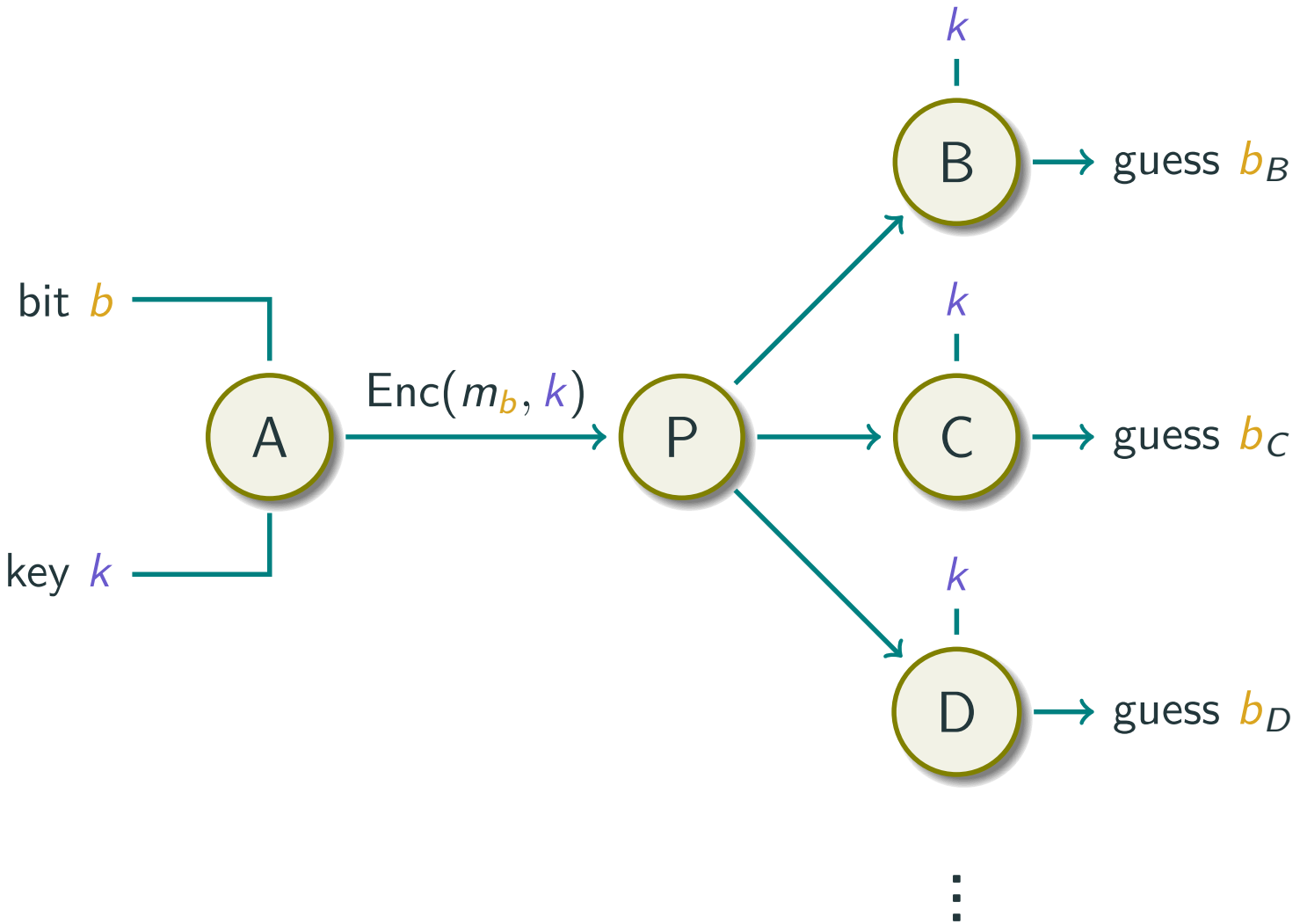
Everlasting security

Unconditional security holds for polynomially many rounds, and everlasting* security for arbitrarily many rounds under pseudorandom unitaries.

*The adversaries are computationally bounded.

s-receiver untelegraphable encryption

The telegraphing attack is extended to s receivers, each given classical information and the key.



Result 5

In analogy with the informational equivalence of no-cloning and no-telegraphing, untelegraphable encryption emerges as a limiting case of uncloneable encryption when the number of receivers grows.

Convergence of cloning to telegraphing attacks

For any uncloneable encryption scheme, the success probability of the cloning attack with s-receiver converges to that of the telegraphing attack at rate

$$\mathcal{O}\left(\frac{1}{\sqrt[3]{s}}\right)$$

Result 6

Minimality of the Haar-measure scheme

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Lower bounds for uncloneable and untelegraphable encryption

For any quantum encryption scheme with ciphertext dimension d, the success probability against cloning and telegraphing attacks is lower bounded by

$$\frac{1}{2} + \Omega\left(\frac{1}{\sqrt{d}}\right).$$

The previous best known lower bound was (Majenz, Schaffner and Tahmasbi, 2021):

$$\frac{1}{2} + \Omega\left(\frac{1}{d}\right).$$

Questions?