# On Quantum Perceptron Learning via Quantum Search

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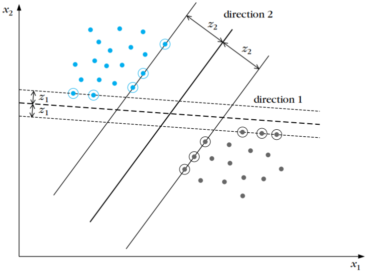
# Machine Learning Task: Binary Linear Classification

## Training dataset

- Training dataset $\mathcal{S}$: $N$ pairs of $\{(\boldsymbol{x}_1, y_1), \ldots, (\boldsymbol{x}_N, y_N)\}$, where training examples $\boldsymbol{x}_{i\in[N]} \in \mathbb{R}^D$, labels $y_{i\in[N]} \in \{+1, -1\}$.




- **Perceptron learning** aims to find a hyperplane $\boldsymbol{w} \in \mathbb{R}^D$, such that $\{\boldsymbol{w} \mid \boldsymbol{w}^T\boldsymbol{x}_i y_i > 0, \forall i \in [N]\}$, and this **feasible set** is called **version space**.

# Dataset Assumption

## Geometric margin

The **geometric margin** $z_h := \min_{i \in [N]} \frac{|\boldsymbol{w}^T \boldsymbol{x}_i + b|}{\|\boldsymbol{w}\|_2}$ of a hyperplane $h = (\boldsymbol{w}, b)$.

Assumption: The maximal margin $\gamma = \max_h z_h \iff \exists\ \boldsymbol{u}^T \boldsymbol{x}_i y_i \geq \gamma$ for all $i \in [N]$.

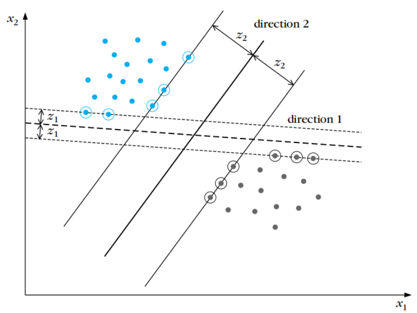



Figure 1: An example of two possible linear classifiers on a linearly separable dataset. Picture from [Theodoridis, S., & Koutroumbas, K. (2006)].

# Query Models

## Classical

A classical *data query* on a training data $(\boldsymbol{x}_i', y_i')$ refers to an evaluation of a Boolean function $f_{\boldsymbol{w}} = \{0, 1\}$, where

$$f_{\boldsymbol{w}}(\boldsymbol{x}_i' y_i') = 1 \iff \boldsymbol{w}^T \boldsymbol{x}_i' y_i' \leq 0 \text{ (misclassified)}. \tag{1}$$

## Quantum

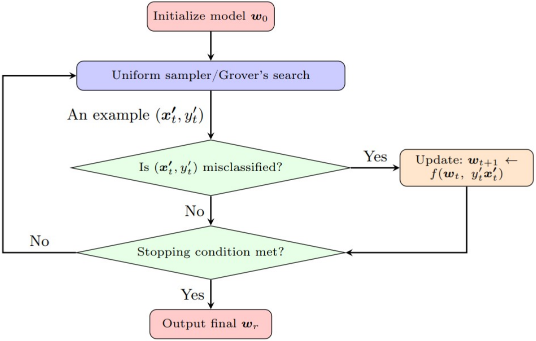
A quantum *data query* on a training data $|\boldsymbol{x}_i' y_i'\rangle$ refers to an evaluation of a quantum oracle $F_{\boldsymbol{w}}$, acting coherently on a superposition of data vectors,

$$F_{\boldsymbol{w}} |\boldsymbol{x}_i' y_i'\rangle = (-1)^{f_{\boldsymbol{w}}(\boldsymbol{x}_i' y_i')} |\boldsymbol{x}_i' y_i'\rangle , \tag{2}$$

or equivalently,

$$F |\boldsymbol{w}\rangle |\boldsymbol{x}_i' y_i'\rangle |0\rangle = |\boldsymbol{w}\rangle |\boldsymbol{x}_i' y_i'\rangle |0 \oplus f_{\boldsymbol{w}}(\boldsymbol{x}_i' y_i')\rangle . \tag{3}$$

- Online perceptron algorithm [Novikoff, 1962]
  Upon misclassification: $\boldsymbol{w} \leftarrow \boldsymbol{w} + \mathbf{x}'_i y'_i$. $\Rightarrow O^*(\frac{N}{\gamma^2})$ [1] classical *data queries*.
- Online quantum perceptron (OQP) [Kapoor et al., 2016]
  Using *Grover's search* [Grover, 1996] for speeding up searching for misclassified examples.
  $\Rightarrow O^*(\frac{\sqrt{N}}{\gamma^2})$ quantum *data queries*.




[1] The asterisk notation $O^*(\cdot)$ to suppress the terms with the error parameter, $\log(D)$, $\log\log(1/\gamma)$ and $\log\log(N)$.

# Previous Work - 2

- Quantum version space perceptron (QVSP) [Kapoor et al., 2016]
  An query to the hyperplane $\boldsymbol{w}_j$ :

$$G_{\mathcal{S}} \left| \boldsymbol{w}_j \right\rangle = (-1)^{1+(f_{\boldsymbol{w}_j}(\boldsymbol{x}_1, y_1) \vee f_{\boldsymbol{w}_j}(\boldsymbol{x}_2, y_2) \vee \cdots \vee f_{\boldsymbol{w}_j}(\boldsymbol{x}_N, y_N))} \left| \boldsymbol{w}_j \right\rangle , \tag{4}$$

  can be built using $O(N)$ quantum *data queries* as Eqn. (2). $\Rightarrow O^*(\frac{N}{\sqrt{\gamma}})$ quantum queries.
- Improvement [Liao et al., 2024]: quantum counting $\Rightarrow O^*(\sqrt{\frac{N}{\gamma}})$ quantum *data queries*.

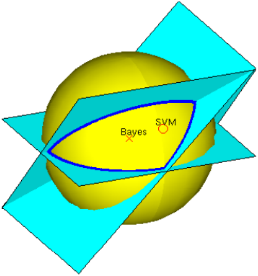



Figure 2: An illustration of version space. Picture from [Minka, T. P. (2001)].

# Result 1 - Classical

## Curse of Dimensionality in QVSP

We examine the QVSP algorithm proposed in [Kapoor et al., 2016], and show that instead of $\Theta(\gamma)$, sampling $\Pr[\boldsymbol{w} \mid \boldsymbol{w} \in \mathcal{VS}] = \Omega(\gamma^D)$ when $\gamma \to 0$.

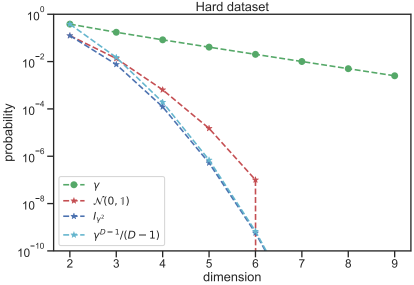



Figure 3: The perfect classification probability-dimension relationship (red) on an artificially designed dataset (Given from [Mohri, M. et al. (2018)].) with its margin $\gamma$ (blue) upper bounded by $\sqrt{1/2^{D-1}}$ (cyan).

- Applying *Grover's search* directly for the classifiers, in the worst case, QVSP uses $O^*(N/\sqrt{\gamma^D})$ instead of $O^*(N/\sqrt{\gamma})$ quantum *data queries*.

# Linear Programming Approaches

Feasibility algorithms: ellipsoid method [Khachiyan, 1979](first polynomial-time solvability), various cutting-plane methods [Vaidya, 1989, Bertsimas and Vempala, 2004].

## Cutting-Plane Random Walk (CP-RW)

Preserving an approximate centroid $\boldsymbol{z}_t$ of the current space $\mathcal{P}_t$.
The half space $H_{t+1} = \{\boldsymbol{w} \mid (\boldsymbol{w} - \boldsymbol{z}_t)^T \boldsymbol{x}'_{t+1} y'_{t+1} > 0\}$ cut by a misclassified example $(\boldsymbol{x}'_{t+1}, y'_{t+1})$.
Update: $\boldsymbol{z}_{t+1} \leftarrow \frac{1}{M} \sum_{j=1}^{M} \boldsymbol{w}'_j$, where $\boldsymbol{w}'_j \sim \mathcal{U}_{H_{t+1}}$. $(\mathcal{P}_{t+1} = H_{t+1} \cap \mathcal{P}_t = H_{t+1} \cap H_t \cap \cdots \cap \mathcal{P}_0)$

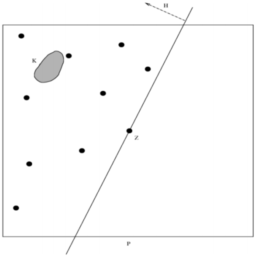



Figure 4: An illustration of the algorithm from [Bertsimas and Vempala, 2004].

- Optimal complexity of $O^*(D \log(1/\gamma))$ rounds (separation oracles).

## *Hit-and-Run* [Smith, 1984]

(1) Choose a line $\ell$ through the current point $\boldsymbol{x}$ uniformly at random.
(2) Move to a point $\boldsymbol{y}$ chosen **uniformly** from $\mathcal{P}_t \cap \ell$.

- The best-known mixing bound: $O^*(D^3)$ steps [Lovász, 1999].

? Quantum speed-ups

## Membership oracles

- Classically, with $t$ queries, $\mathcal{O}_{\mathcal{P}_t}(\boldsymbol{w}) = 0$ iff $\boldsymbol{w} \in \mathcal{P}_t$. where $f_{\boldsymbol{w}}(\boldsymbol{x}_j' y_j', \boldsymbol{z}_{j-1}) = 0$ iff $(\boldsymbol{w} - \boldsymbol{z}_{j-1})^T \boldsymbol{x}_j' y_j' > 0$, $\forall j \in [t]$.
- Quantumly, $\mathcal{Q}_{\mathcal{P}_t}$ can be implemented using $O(\sqrt{t})$ quantum *data queries* [Liao et al., 2024], with

$$P_t \left|\boldsymbol{w}\right\rangle \left|\boldsymbol{x}_j' y_j', \boldsymbol{z}_{j-1}\right\rangle \left|0\right\rangle = \left|\boldsymbol{w}\right\rangle \left|\boldsymbol{x}_j' y_j', \boldsymbol{z}_{j-1}\right\rangle \left|0 \oplus f_{\boldsymbol{w}}(\boldsymbol{x}_j' y_j', \boldsymbol{z}_{j-1})\right\rangle, \forall j \in [t+1]. \tag{5}$$

■ $t \leq O^*(D \log(1/\gamma)) \Rightarrow$ Naive Hybrid cutting-plane random walk (HCP-RW) algorithm.

? Further quantum speed-ups- sampling? [Wocjan and Abeyesinghe, 2008]

## Unique eigenvector

It was shown by [Szegedy, 2004, Magniez et al., 2007] that for an *irreducible* and time-reversible Markov chain $P$ with stationary distribution $\boldsymbol{\pi}$. The quantum walk $W(P) := R_{\mathcal{B}} R_{\mathcal{A}}$, where $R_{\mathcal{I}} = 2\,\Pi_{\mathcal{I}} - \mathbb{1}$, and $\mathcal{A} = \textbf{span}(|x\rangle\,|0\rangle : x \in \Omega)$, $\mathcal{B} = \textbf{span}(|0\rangle\,|x\rangle : x \in \Omega)$, has a unique eigenvalue-1 eigenvector

$$|\pi\rangle = |\pi\rangle'\,|0\rangle = |\pi\rangle' = \sum_{x \in \Omega} \sqrt{\pi_x}\,|x\rangle\,|0\rangle\,.$$

## Quantum walk operator

Implementation $W_t := W(P_t) = U(P_t)^\dagger S U(P_t) R_{\mathcal{A}} U(P_t)^\dagger S U(P_t) R_{\mathcal{A}}$. With the swap $S$, and quantum walk update $U(P_t)$ :

$$U(P_t)\,|x\rangle\,|0\rangle = |x\rangle\,|p_x\rangle = |x\rangle \sum_{y \in \Omega} \sqrt{p_{xy}}\,|y\rangle\,, \ \forall \boldsymbol{x} \in \Omega,$$

where $p_{xy}$ denotes the transition probability from $x$ to $y$.

# Result 2 - Quantum

## Fully-Quantum Cutting-Plane Quantum Walk Algorithm (QCP-QW)

- **High-level:**
  Outer loop: CP method, performing $O^*(D\log(1/\gamma))$ updates to prepare final solution $|\pi_r\rangle$.
  Inner loop: *Grover's search* to amplify the probability of finding a separating hyperplane.

- **Middle-level:**
  Applying $O^*(D)$ non-destructive quantum mean estimation [a] $U_t^{\text{mean}}$ on $|\tilde{\pi}_t\rangle$ to acquire the multivariate centroid $\boldsymbol{z}_{t+1}$ of $\tilde{\boldsymbol{\pi}}_{t+1}$.
  Similarly, applying $O^*(D)$ non-destructive affine estimation $U_t^{\text{aff}}$ on $|\tilde{\pi}_t\rangle$ to obtain the affine transformation matrix $S_{t+1}$ of $\tilde{\boldsymbol{\pi}}_{t+1}$.

- **Low-level:**
  Applying the affine transformation $g_t$ to the states $|\tilde{\pi}_t\rangle$ to ensure the convex body $\mathcal{P}_t$ is in near-isotropic position.
  Applying the quantum walk evolution $\tilde{U}_{t,m}$ to generate the distribution $|\tilde{\pi}_{t+1}\rangle$ via

$$|g_t\tilde{\pi}_{t+1}\rangle = \tilde{U}_{t,m}\,|g_t\tilde{\pi}_t\rangle\,.$$

[a] For non-destructive multidimensional quantum amplitude estimation circuits, we point out the possibility to do so from [Harrow and Wei, 2020, Cornelissen et al., 2022, Cornelissen and Hamoudi, 2023, van Apeldoorn et al., 2023].

# Quantum Walk Evolution Operator $U_{t,m}$

## Grover's $\frac{\pi}{3}$ fix point search [Grover, 2005]

Two arbitrary quantum states with overlap $|\langle\pi_t|\pi_{t+1}\rangle|^2 \geq p$ $(0 < p \leq 1)$.
Given $|\pi_t\rangle$, $|\tilde{\pi}_{t+1}\rangle = U_{t,m}|\pi_t\rangle$, such that $\||\tilde{\pi}_{t+1}\rangle - |\pi_{t+1}\rangle\| \leq \epsilon_1 > 0$.
Choosing $m = O(\log(\log 1/\epsilon_1))$, unitaries $\{R_t, R_t^\dagger, R_{t+1}, R_{t+1}^\dagger\}$ are invoked at most $O(\log 1/\epsilon_1)$ times.

## Recursive quantum walk evolutions

$U_{t,0} = \mathbb{1}$, and $U_{t,i+1} = U_{t,i} \cdot R_t \cdot U_{t,i}^\dagger \cdot R_{t+1} \cdot U_{t,i}$.

The unitaries $R_t$, $R_{t+1}$ operators are selective phase shifts that $R_t = e^{\frac{i\pi}{3}}\Pi_t + \Pi_t^\perp$, where $\Pi_t$ is the orthogonal projector onto **span** $(|\pi_t\rangle)$.

## Example

$$U_{t,1} = U_{t,0} \cdot R_t \cdot U_{t,0}^\dagger \cdot R_{t+1} \cdot U_{t,0} = R_t \cdot R_{t+1}$$
$$U_{t,2} = U_{t,1} \cdot R_t \cdot U_{t,1}^\dagger \cdot R_{t+1} \cdot U_{t,1} = (R_t \cdot R_{t+1}) \cdot R_t \cdot (R_{t+1}^\dagger R_t^\dagger) \cdot R_{t+1} \cdot (R_t \cdot R_{t+1})$$
$$\dots$$
$$|\tilde{\pi}_{t+1}\rangle = U_{t,m}|\pi_t\rangle = U_{t,m-1} R_t U_{t,m-1}^\dagger R_{t+1} U_{t,m-1}|\pi_t\rangle = R_t \cdot R_{t+1} \cdots R_t \cdot R_{t+1}|\pi_t\rangle$$

## Example

The state $|\pi_t\rangle$ is a uniform superposition of states on the convex body $\mathcal{P}_t$. It can be decomposed in the subspace $\mathcal{H}_{\pi_t} = \textbf{span}\,(|\pi_{t+1}\rangle, |\pi_{t+1}^{\perp}\rangle)$ as $|\pi_t\rangle = \sin\theta_{t+1}|\pi_{t+1}\rangle + \cos\theta_{t+1}|\pi_{t+1}^{\perp}\rangle$, where

$$|\pi_{t+1}\rangle = \frac{\Pi_{\mathcal{P}_{t+1}}|\pi_t\rangle}{\|\Pi_{\mathcal{P}_{t+1}}|\pi_t\rangle\|} = \frac{\sum_{x\in\mathcal{P}_{t+1}}|x\rangle\langle x|\sum_{x'\in\mathcal{P}_t}\sqrt{\pi_{t,x'}}|x'\rangle|0\rangle}{\sqrt{\langle\pi_t|\Pi_{\mathcal{P}_{t+1}}|\pi_t\rangle}} = \frac{\sum_{x\in\mathcal{P}_{t+1}}\sqrt{\pi_{t,x}}|x\rangle|0\rangle}{\sqrt{\sum_{x\in\mathcal{P}_{t+1}}\pi_{t,x}}},$$

From the actions of quantum oracle $P_{t+1}$ in Eqn. (5):

$$P_{t+1}|\boldsymbol{w}\rangle|\boldsymbol{x}'_{t+1}y'_{t+1},\boldsymbol{z}_t\rangle|0\rangle = |\boldsymbol{w}\rangle|\boldsymbol{x}'_{t+1}y'_{t+1},\boldsymbol{z}_t\rangle|0\oplus f_{\boldsymbol{w}}(\boldsymbol{x}'_{t+1}y'_{t+1},\boldsymbol{z}_t)\rangle,$$

it can be easily verified that $P_{t+1}|\pi_{t+1}\rangle|0\rangle = |\pi_{t+1}\rangle|0\rangle$ and $P_{t+1}|\pi_{t+1}^{\perp}\rangle|0\rangle = |\pi_{t+1}^{\perp}\rangle|1\rangle$. Herewith, $R_{t+1} = e^{\frac{i\pi}{3}}\Pi_{t+1} + \Pi_{t+1}^{\perp}$ can be constructed by letting

$$R_{t+1} = P_{t+1}^{\dagger}(\mathbb{1}\otimes(e^{\frac{i\pi}{3}}|0\rangle\langle 0| + |1\rangle\langle 1|))P_{t+1}. \tag{6}$$

As has been shown in [Magniez et al., 2007] and [Wocjan and Abeyesinghe, 2008, Corollary 2], the phase estimation [Cleve et al., 1998] circuit $PE(W_t)$ can be leveraged to realise $R_t$.
For some $\epsilon_2 > 0$, there is a quantum circuit $V_t$ of $\mathbb{C}^d \otimes \mathbb{C}^d \otimes (\mathbb{C}^2)^{\otimes ac}$, using $PE(W_t)$ circuit $c = O(\log \frac{1}{\sqrt{\epsilon_2}}))$ times with $a = O(\log \frac{1}{\Delta})$ ancillary qubits each.

$$\tilde{R}_t = V_t^\dagger (\mathbb{1} \otimes (e^{\frac{i\pi}{3}} |0\rangle \langle 0|^{\otimes ac} + (\mathbb{1} - |0\rangle \langle 0|^{\otimes ac})))V_t, \tag{7}$$

that $\| \tilde{R}_t |\psi_k^\pm\rangle |0\rangle^{\otimes ac} - R_t |\psi_k^\pm\rangle |0\rangle^{\otimes ac} \| \leq 2\sqrt{\epsilon_2}$.

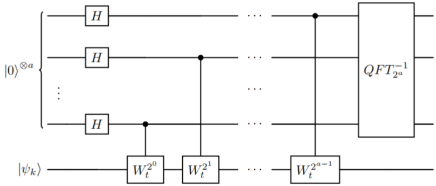



Figure 5: Phase estimation circuit $PE(W_t)$, where $H$ is the Hadamard gate, $QFT^{-1}$ denotes the inverse quantum Fourier transform circuit, and $W_t = U(P_t)^\dagger SU(P_t)R_{\mathcal{A}}U(P_t)^\dagger SU(P_t)R_{\mathcal{A}}$

# The Main Theorem

## Theorem (Hit-and-Run Quantum Walk)

*Let $P_t$ be an ergodic symmetric Markov chain defined by the* **Hit-and-Run** *algorithm with stationary uniform distribution $\boldsymbol{\pi}_t$ on the support $\mathcal{P}_t$ with an initial distribution $|\pi_t\rangle = |\pi'_t\rangle\,|0\rangle$. If the following properties hold:*

- **Mixing time:** $d_{\mathrm{TV}}(P_t^{\tau} \cdot \boldsymbol{\pi}_{t+1}, \boldsymbol{\pi}_t) \leq \epsilon$.
- **Warmness:** $\|\boldsymbol{\pi}_{t+1}/\boldsymbol{\pi}_t\| = O(1)$, *where* $\|\boldsymbol{\pi}/\boldsymbol{\sigma}\| := \int_{\mathbb{R}^D} \frac{\pi(\boldsymbol{x})}{\sigma(\boldsymbol{x})}\pi(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}$.
- **Overlap:** $|\langle \pi_t|\pi_{t+1}\rangle| = \int_{\mathbb{R}^D} \sqrt{\pi_t(\boldsymbol{x})\pi_{t+1}(\boldsymbol{x})}\,d\boldsymbol{x} = \Omega(1)$.

*Then, we can obtain $|\tilde{\pi}_{t+1}\rangle = \tilde{U}_{t,m}\,|\pi_t\rangle$ with $\|\,|\tilde{\pi}_{t+1}\rangle - |\pi_{t+1}\rangle\,\| \leq \epsilon$, using $O\big(\sqrt{\tau}\log(1/\epsilon)\big)$ calls to the quantum walk operators $W_t$.*

- Using *Hit-and-Run*, start from $\boldsymbol{\pi}_{t+1}$, takes $\tau = O^*(D^3)$ steps to generate a uniform distribution $\boldsymbol{\pi}_t$. The quantum walk evolution $\Rightarrow$ $U_{t,m}$ can be achieved by using $O^*\left(D^{1.5}\right)$ calls to the quantum walk operators $W_t$.

# Lemmas

For uniform $\boldsymbol{\pi}_t$ over $\mathcal{P}_t$, its quantum sample in the continuous space
$|\pi_t\rangle' = \int_{\mathcal{P}_t} \sqrt{\pi_{t,x}}\,|x\rangle\; dx = \int_{\mathcal{P}_t} \sqrt{\frac{1}{V(\mathcal{P}_t)}}\,|x\rangle\; dx.$

## Overlap lemma

From [Bertsimas and Vempala, 2004], the volume left in each round is at least 1/3 with high probability.

$$|\langle \pi_t | \pi_{t+1}\rangle| = \int_{\mathbb{R}^D} \sqrt{\frac{1}{V(\mathcal{P}_t)V(\mathcal{P}_{t+1})}}\mathrm{d}\boldsymbol{x} = \frac{V(\mathcal{P}_{t+1})}{\sqrt{V(\mathcal{P}_t)V(\mathcal{P}_{t+1})}} \geq \sqrt{\frac{1}{3}} = \Omega(1).$$

Hence, in QCP-QW algorithm, the overlap condition $|\langle \pi_t | \pi_{t+1}\rangle| = \Omega(1)$ is met.

## Warmness lemma

$$\|\boldsymbol{\pi}_{t+1}/\boldsymbol{\pi}_t\| = \int_{\mathbb{R}^D} \frac{\pi_{t+1}(\boldsymbol{x})}{\pi_t(\boldsymbol{x})}\pi_{t+1}(\boldsymbol{x})\mathrm{d}\boldsymbol{x} = \int_{\mathcal{P}_{t+1}} \frac{\frac{1}{V(\mathcal{P}_{t+1})}}{\frac{1}{V(\mathcal{P}_t)}}\frac{1}{V(\mathcal{P}_{t+1})}\mathrm{d}\boldsymbol{x} = \frac{V(\mathcal{P}_t)}{V(\mathcal{P}_{t+1})} \leq 3 = O(1).$$

Hence, in QCP-QW algorithm, the warmness conditions $\|\boldsymbol{\pi}_{t+1}/\boldsymbol{\pi}_t\| = O(1)$ is met.

## A summary of algorithms and *data query* complexities

| Algorithm | Complexity |
|---|---|
| Version Space Perceptron | $O^*(D^{\frac{1}{2}}\frac{N}{\gamma^D})$ |
| QVSP | $O^*(D^{\frac{1}{4}}\sqrt{\frac{N}{\gamma^D}})^{\mathrm{a}}$ |
| Online Perceptron | $O^*(\frac{1}{\gamma^2}\log(1/\gamma^2)N)$ |
| OQP | $O^*(\frac{1}{\gamma^2}\log(1/\gamma^2)\sqrt{N})$ |
| CP-RW (Hit-and-Run) | $O^*(D\log(\frac{1}{\gamma})(N+D^5\log(\frac{1}{\gamma})))^{\mathrm{b}}$ |
| HCP-RW (1) | $O^*(D\log(\frac{1}{\gamma})(\sqrt{N}+D^{4.5}\sqrt{\log(\frac{1}{\gamma})}))$ $^{\mathrm{b}}$ |
| QCP-QW (2) | $O^*(D\log(\frac{1}{\gamma})(\sqrt{N}+D^3\sqrt{\log(\frac{1}{\gamma})}))$ |

[a] Here we only present the corrected result of the improved-QVSP of Liao et al. (2024).

[b] For Hit-and-Run CP-RW and HCP-RW introduced in Section 4.2, we simplify the result using $O(\log^2 N)\le O(D)$. Otherwise, tighter bounds can be achieved by replacing an $O^*(D)$ factor in $O^*(D^5)$ and $O^*(D^{4.5})$ with $\log^2 N$.

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Thanks for your attention.

Figure 6: https://arxiv.org/abs/2503.17308